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Hydrodynamic characteristics of rectangular submerged breakwaters

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Abstract

In this study, method of separation of variables is applied for investigating two dimensional submerged breakwaters characteristics in an infinite domain. For this purpose, the fluid domain is divided into discrete sub-domains and the velocity potentials are approximated using infinite series of orthogonal functions in each sub-domain. Expression for each region is developed according to boundary conditions except those at common boundaries between the regions. Continuity of pressure and normal velocity, which demonstrate equality of pressure and normal velocity in the boundaries, are satisfied in order to obtain the unknown coefficients in the series. A classical rectangular cross section in finite water depth with four sub-domains is studied in three degrees of freedom e.g. sway, heave and roll. Results are in good agreement with other researches as well as numerical method using boundary element software ANSYS AQWA which is used as a backup in this study. Furthermore, the effects of submergence depth on the exiting forces and the transmission and reflection coefficients are studied. The results show with increasing the submergence depth, reflection coefficient decreases and transmission coefficient increases. Also, it is shown that with enhancing the submergence depth, maximum horizontal and vertical exiting forces decrease and exiting moment increases smoothly.

Keywords: Separation of variables; Submerged breakwater; Rectangular cross section; Transmission coefficient; Reflection coefficient.

1. Introduction

Development of coastal or inland waters may often depend on sea behavior at a specific site and breakwaters of various dimensions and configurations have been widely employed to increase the use of locations exposed to wave attack. The purpose of installing a breakwater is to reduce the height of incident waves to an acceptable level in respect to the intended use of the site. Increasing the number of pleasure vessels and related companies has accrued

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a demand for more protected sites. Fixed breakwater has been widely used to weaken incident coastal waves. Economics and degree of wave protection often dictate possible breakwater options. Floating breakwaters are not only more economical, but also are more environmentally friendly devices which can be used instead of fixed breakwaters. A floating breakwater usually consists of a floating box with finite draft which exposed to incident waves. For two dimensional problems the movement of the floating breakwater can be limited to three degrees of freedom such as sway, heave, and roll.

There are three methods which have been applied to study floating structures in different conditions: numerical methods, experimental methods, and analytical methods. In framework of numerical methods, finite element and boundary element methods are two popular and effective approaches which have been widely applied to floating body problems. Examples of numerical methods are using finite- infinite element method by Li *et al*. (1991), using boundary element method for studying floating structures by Yamamoto *et al*. (1980), using boundary element method for analyzing various cross sections of floating breakwater by Masoudi and Zeraatgar (2017) and using finite element method for studying a moored floating breakwater by Tabatabaei and Zeraatgar (2018). Because of drawbacks of both finite element and boundary element methods, researchers became interested in coupled finite element and boundary element method. For instance Wu and Taylor (2003) used this new method studying a submerged elliptical cylinder motions. Also, Sannasiraj *et al*. (1995) is a good example for experimental method which mooring forces and motions have been discussed for a pontoon type breakwater. Ji *et al*. (2018) is another new experimental study in which double-row floating breakwaters were analyzed.

Analytical method in studying floating structures, firstly was developed by Garrett (1971), in which wave forces on a dock was calculated. Other researchers such as Helm (1982), Wu and Taylor (1990), Berggren and Johansson (1992), Lee (1995), Hsu and Wu (1997), Zheng *et al*. (2004a), Zheng *et al*. (2004b), Masoudi and Zeraatgar (2016) also used analytical method to investigate floating structures problem. Analytical solution consists of separating the domain to sub-domains and the velocity potentials in each sub-domain approximated by orthogonal functions. After satisfying the boundary conditions and the common boundaries between sub-domains, the unknown coefficients in orthogonal functions are obtained and the velocity potentials become implicit in each sub-domain. Having determined the velocity potentials and wave characteristics in both sides of the body, the transmission and reflection coefficients are obtained.

In this study, a rectangular section breakwater submerged in water of finite depth and infinite extent is analyzed using the separation of the variables method in regular sinusoidal waves. After obtaining the radiation potentials in three degrees of freedom e.g. sway, heave and roll, added mass and damping coefficients of sway, heave and roll motions are obtained. Diffraction problem is solved according to linear wave theory and resulting forces on body for three degrees of freedom are obtained. Having known the diffraction potential in each region, the transmission and reflection coefficients are obtained. Furthermore, a parametric study on effects of submergence depth on wave exciting forces, transmission and reflection coefficients carried out. Also, the effect of submergence depth on maximum exciting forces on the structure is discussed in the following sections.

2. Methods and materials

For large ratio of the breakwater length to the wavelength, the fluid is assumed incompressible and irrotational. According to Berggren and Johansson (1992), there will be a scalar function which known as velocity potential, ϕ, that satisfies the Laplace equation as shown in Equation (1). Also the velocity components and pressure can be obtained using Equations (2) and (3).

$$\nabla^2\phi = 0 \tag{1}$$

$$\frac{\partial\phi}{\partial x} = u \qquad \frac{\partial\phi}{\partial y} = v \qquad \frac{\partial\phi}{\partial z} = w \tag{2}$$

$$P = -\rho\left(\frac{\partial\phi}{\partial t} + \frac{1}{2}\nabla\phi^2 + gz\right) + c(t) \tag{3}$$

In which u, v, w are velocities in x, y, z directions respectively. Also P is pressure and *c(t)* is a constant. Basic configuration of the breakwater and the arrangement of coordinate system are shown in Figure 1. It is assumed that a linear wave with height of H and circular frequency of $\omega = \frac{2\pi}{T}$ propagates to the positive x direction with θ angle. The total potential of the problem can be separated in three parts. First one is the incident wave potential of linear waves which considered independently from existence of the breakwater given by (Zheng *et al*. 2007):

$$\varphi_I = -\frac{igA}{\omega}\frac{\cosh[k(z+h_1)]}{\cosh(kh_1)}\exp(ikx\cos\theta) \tag{4}$$

$$\omega^2 = \mathrm{gk}\tanh kh_1 \tag{5}$$

Second one is the diffraction potential (ϕ_d), which induced by interaction of incident wave and the breakwater, and the third is the induced potential from motions of the structure in three degrees of freedom which known as radiation potentials (ϕ_r).

Referring to Figure 1, the problem is considered as a 2D case, motions are restricted in three degrees of freedom which are heave, sway and roll as specified with indices 1, 2 and 3, respectively. The total potential, ϕ_t , expressed as follows:

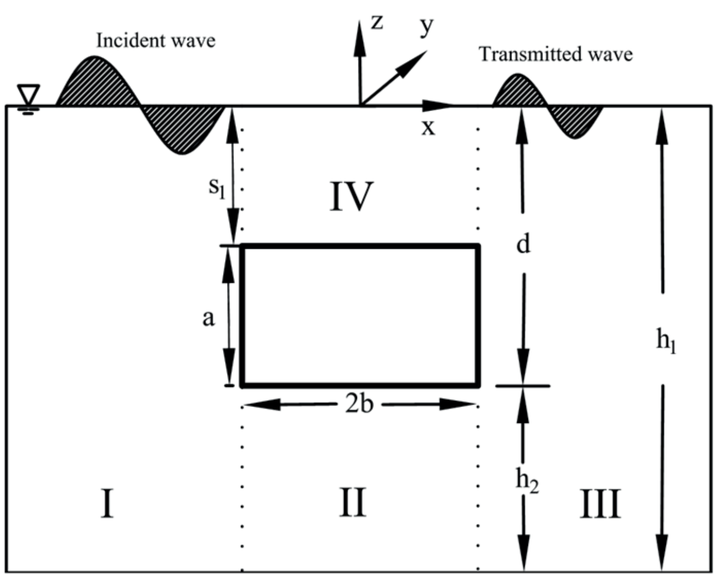


Figure 1. Basic configuraion, coordinate system and dividing the domain into four subdomains

$$\phi_t = \phi_i + \phi_d + \sum_{l=1}^{3} \phi_r^L \tag{6}$$

where, L refers to the number that assigned to the motions and ϕ_r^L is the radiation potential of L[th] motion. The unknown terms in above equation are ϕ_d and ϕ_r^L which will be addressed in following subsections.

2.1. Diffraction problem

The linear diffraction problem and its boundary conditions can be expressed by oscillatory function ϕ_d, given by (Zheng *et al*. 2007):

$$\phi_d(x,z,y) = \varphi_d(x,z)\exp(ikysin\,\theta) \tag{7}$$

$$\frac{\partial \varphi_d}{\partial z} - \frac{\omega^2}{g}\varphi_d = 0 \ \ (z = 0\,) \tag{8}$$

$$\frac{\partial \varphi_d}{\partial z} = 0 \ \ (z = -h_1) \tag{9}$$

$$\frac{\partial \varphi_d}{\partial n} = -\frac{\partial \varphi_i}{\partial n} \ \ (on\ S_0) \tag{10}$$

$$\lim_{x\to\infty}\left[\frac{\partial \varphi_d}{\partial x} \pm ikcos\theta\varphi_d\right] = 0 \tag{11}$$

The boundary value problem for the diffraction potential is defined by governing Laplace equation and the boundary conditions are defined in Equations (8) to (11), where n is the unit normal vector outward the body and S_0 is the wetted surface of the breakwater.

2.2. Radiation problem

In the framework of the linear theory, the radiation problem and its boundary conditions can also be described by the following oscillatory radiation potential and boundary conditions (Zheng *et al*. 2007):

$$\phi_r^L(x,z,y) = -i\omega A_r^L\varphi_r^L(x,z)\exp(ikysin\,\theta) \tag{12}$$

$$\frac{\partial \varphi_r^L}{\partial z} - \frac{\omega^2}{g}\varphi_r^L = 0 \ \ (z = 0\,) \tag{13}$$

$$\frac{\partial \varphi_r^L}{\partial z} = 0 \ \ (z = -h_1) \tag{14}$$

$$\frac{\partial \varphi_r^L}{\partial z} = \delta_{1,L} - (x - x_0)\delta_{3,L} \ \ (z = -s_1\ or\ z = -d\,,\ |x| \le b) \tag{15}$$

$$\frac{\partial \varphi_r^L}{\partial x} = \delta_{2,L} + (z - z_0)\delta_{3,L}\ (-d \le z \le -s_1\,,\ |x| = b) \tag{16}$$

$$\lim_{x\to\infty}\left[\frac{\partial \varphi_r^L}{\partial x} \pm ikcos\theta\varphi_r^L\right] = 0 \tag{17}$$

$$\delta_{i,j} = \begin{cases}1 & i = j\\ 0 & i \ne j\end{cases} \tag{18}$$

The amplitude of L[th] motion of the body is denoted by A_r^L and (x_0, z_0) is the center of rotation, which is the center of gravity of the body. The boundary value problem for the linear radiation potential can be defined by governing Laplace equation and the boundary condition as defined in Equations (13) to (17).

2.3. Analytical solution

Considering Figure 1 as the basic configuration of the two dimensional submerged breakwater problem, the domain is divided into four sub-domains denoted with I, II, III and IV. Applying the separation of variables method gives the complex spatial potentials in each sub-domain expressed in terms of orthogonal series as below (Zheng *et al*. 2007). For diffraction problem velocity potentials are given in Equations (19) to (22) for regions I to IV respectively:

$$\varphi_{d1} = \sum_{n=1}^{\infty} A'_{1n} e^{-\gamma_n(x-b)} \cos[\lambda_n(z+h_1)] \quad (19)$$

$$\varphi_{d2} = -\varphi_i + \sum_{n=1}^{\infty} \left[A'_{2n} e^{\mu_n(x+b)} + B'_{2n} e^{-\mu_n(x-b)}\right] \cos[\beta_n(z+h_1)] \quad (20)$$

$$\varphi_{d3} = \sum_{n=1}^{\infty} A'_{3n} e^{\gamma_n(x+b)} \cos[\lambda_n(z+h_1)] \quad (21)$$

$$\varphi_{d4} = -\varphi_i + \sum_{n=1}^{\infty} \left[A'_{4n} e^{\upsilon_n(x+b)} + B'_{4n} e^{-\upsilon_n(x-b)}\right] \cos[\alpha_n(z+s_1)] \quad (22)$$

For radiation problem velocity potentials are given in Equations (23) to (26) for regions I to

$$\varphi_{r1}^{L} = \sum_{n=1}^{\infty} A_{1n}^{L} e^{-\gamma_n(x-b)} \cos[\lambda_n(z+h_1)] \quad (23)$$

$$\varphi_{r2}^{\mathrm{L}} = \varphi_{r2P}^{L} + \sum_{n=1}^{\infty} \left[A_{2n}^{(\mathrm{L})} e^{\mu_n(x+b)} + B_{2n}^{(\mathrm{L})} e^{-\mu_n(x-b)}\right] \cos[\beta_n(z+h_1)] \quad (24)$$

$$\varphi_{r3}^{L} = \sum_{n=1}^{\infty} A_{3n}^{L} e^{\gamma_n(x+b)} \cos[\lambda_n(z+h_1)] \quad (25)$$

$$\varphi_{r4}^{L} = \varphi_{r4P}^{L} + \textstyle\sum_{n=1}^{\infty} \left[A_{4n}^{(L)} e^{\upsilon_n(x+b)} + B_{4n}^{(L)} e^{-\upsilon_n(x-b)}\right] \cos[\alpha_n(z+s_1)] \quad (26)$$

In Equations (19) to (26) Eigen values $(\gamma_n, \mu_n, \beta_n, \lambda_n, \upsilon_n, \alpha_n)$ are given by:

$$\lambda_1 = -ik, ktanh(kh_1) = \frac{\omega^2}{g} \quad n = 1 \quad (27)$$

$$\lambda_n tan(\lambda_n h_1) = \frac{-\omega^2}{g} \quad n = 2,3,\ldots \quad (28)$$

$$\alpha_1 = -ik_1,\ k_1 tanh(k_1 s_1) = \frac{\omega^2}{g} \quad n = 1 \quad (29)$$

$$\alpha_n tan(\alpha_n s_1) = \frac{-\omega^2}{g} \quad n = 2,3,\ldots \quad (30)$$

$$\beta_n = \frac{(n-1)\pi}{(h_1 - d)} \quad n = 1,2,3,\ldots \quad (31)$$

$$\upsilon_n = \begin{cases} -i\sqrt{k_1^2 - k_0^2} & n = 1 \\ \sqrt{\alpha_n^2 + k_0^2} & n = 2,3,\ldots \end{cases}, \ where\ k_0 = ksin\theta \quad (32)$$

$$\gamma_n = \begin{cases} -ikcos\theta & n = 1 \\ \sqrt{\lambda_n^2 + k_0^2} & n = 2,3,\ldots \end{cases} \quad (33)$$

$$\mu_n = \begin{cases} k_0 & n = 1 \\ \sqrt{\beta_n^2 + k_0^2} & n = 2,3,\ldots \end{cases} \quad (34)$$

Furthermore, in Equation (24) and (26), φ_{r2P}^{L} and φ_{r4P}^{L} are particular solutions for L[th] radiation motions sub-domain II and IV, respectively (Zheng *et al.* 2007):

$$\varphi_{r2P}^{(L)} = C_{F2}(z)\left[\delta_{1,L} - (x - x_0)\delta_{3,L}\right] \quad (35)$$

$$\varphi_{r4P}^{(L)} = C_{F4}(z)\left[\delta_{1,L} - (x - x_0)\delta_{3,L}\right] \quad (36)$$

$$C_{F2}(z) = \frac{\cosh[\mu_1(z+h_1)]}{\mu_1 \sinh(\mu_1 h_2)} \quad (37)$$

$$C_{F4}(z) = \frac{\frac{\omega^2}{g}\sinh(k_0 z) + k_0\cosh(k_0 z)}{k_0\left[\frac{\omega^2}{g}\cosh(k_0 s_1) - k_0\sinh(k_0 s_1)\right]} \quad (38)$$

The potentials given above describe the fluid in each respective region and satisfy all boundary conditions except common boundaries between the regions. Now, the problem is to calculate unknown coefficients $A_{1n}^{(L)}$ ‘$A_{2n}^{(L)}$ ‘$A_{3n}^{(L)}$ ‘$A_{4n}^{(L)}$ $B_{4n}^{(L)}$ $B_{2n}^{(L)}$‘ for radiation problem and $A_{1n}^{'}$ ‘$A_{2n}^{'}$‘$A_{3n}^{'}$, $A_{4n}^{'}$, $B_{2n}^{'}$ and $B_{4n}^{'}$ for diffraction problem in the series. It should be noted that each coefficient has a unit which depends on the respective motion in radiation problem. These coefficients are found by imposing the boundary conditions at the common boundaries between the regions which are the continuity of pressure and normal velocity. In mathematical terms, it means equal potentials and their normal derivatives at boundaries. The detail of this method is discussed by Masoudi and Zeraatgar (2016).

In order to find the solution, the infinite series of orthogonal functions must be truncated. If N assumed to be the number of the considered orthogonal functions in the series, then it will be a system of 6N complex equations and equal numbers of unknown coefficients. Organizing the coefficient in the matrices gives:

$$S.X = F \tag{39}$$

where, X is the unknown coefficients vector. There are three radiations and diffraction potentials by Equation (39). It should be noted that F is obtained from the boundary conditions. So, for each of four problems X is obtained. Finally, imposing the coefficients in Equations (19) to (26), the velocity potentials for each region will be obtained.

3. Results and Discussion

3.1. Expressions for hydrodynamic coefficients

The hydrodynamic coefficients including added mass $m_{L,j}$ and damping coefficient $N_{L,j}$ are defined here by:

$$m_{L,j} = \rho \int_{S_0} Re(\varphi_r^L) n_j ds \tag{40}$$

$$N_{L,j} = \rho \int_{S_0} Im(\varphi_r^L) n_j ds \tag{41}$$

Also, C_{mj} and C_{dj} are non-dimensional added mass and damping coefficients given by:

$$C_{mj} = \begin{cases} \dfrac{m_{j,j}}{2\rho b \mathrm{d}} & i = 1,2 \\ \dfrac{m_{j,j}}{2\rho b^3 \mathrm{d}} & i = 3 \end{cases} \tag{42}$$

$$C_{dj} = \begin{cases} \dfrac{N_{j,j}}{2\rho\omega b \mathrm{d}} & i = 1,2 \\ \dfrac{N_{j,j}}{2\rho\omega b^3 \mathrm{d}} & i = 3 \end{cases} \tag{43}$$

3.2. Expressions for transmission and reflection coefficients

Transmission coefficient (T_w) is defined as the amplitude of transmitted wave to the amplitude of the incident wave. Also reflection coefficient (R_w) is defined as the amplitude of reflected wave to the amplitude of the incident wave. If breakwaters assumed to be stationary, using linear Bernoulli equation, one can calculate transmission and reflections coefficients as follows:

$$T_w = \frac{\left| i\omega A_{31}^{'} \cosh(kh_1) \right|}{gA} \tag{44}$$

$$R_w = \left| 1 + \frac{i\omega A_{11}^{'} \cosh(kh_1)}{gAe^{ikbcos\theta}} \right| \tag{45}$$

Longuet–Higgins (1977) proposed the horizontal drift force (F_d) directly in terms of the reflection coefficient as follows:

$$F_d = \frac{Ec_g}{c}(1 + R_w^2 - T_w^2) = \frac{2Ec_g}{c}R_w^2 \qquad (46)$$

In which c_g is wave group velocity, c is phase velocity, $E = \frac{1}{2}\rho g A^2$ is wave energy, A is amplitude of incident wave, T_w is transmission coefficient, and R_w is reflection coefficient.

3.3. Computer code validation

Based on the formulation and what discussed in section 2, a computer code in MATLAB ® software is developed. Main inputs of this computer code are θ, a, b, h_1, h_2, s_1, d, A and number of truncated sentences in the orthogonal series N.

Outputs of this code are hydrodynamic characteristics of the domain. In order to verify the computer code, a rectangular submerged breakwater of $\frac{s_1}{h_1} = 0.2, \frac{a}{h_1} = 0.2, \frac{h_1}{b} = 6, \theta = 30^0$ is considered.

Figure 2 demonstrates the added mass (C_{mj}), damping coefficients (C_{dj}) of the proposed breakwater. The results of this study patent are dimensionless, and are quite comparable with the study by Zheng *et al*. (2007). Also, present numerical study, considering some difference which is a result of 3D to 2D simulation; demonstrate acceptable trends and values in comparison with analytical study.

3.4. Parametric study

In order to accomplish the effect of submergence depth on exciting forces, reflection and transmission coefficients, the non-dimensional submergence depth, $\frac{s_1}{h_1}$ is considered. Figure 3 shows the effect of non-dimensional submergence depth on exciting forces in three degrees of freedom for $\frac{a}{h_1} = 0.2, \frac{h_1}{b} = 6$, using analytical (a, c, e) and numerical (b, d ,f) method. Also, to avoid the singularity of $\theta = 0$, in analytical method, the incident wave angle of $\frac{\pi}{180}$ is considered. From Figure 3, it can be concluded that increasing the submergence depth results in decreasing the exciting of heave, sway, and roll motions. Furthermore, the effect of submergence parameter on maximum forces and moments on the structure is shown in Figure 3(g). As it can be seen, increasing the submergence depth is caused decreasing in both horizontal and vertical maximum forces on the structure, but the maximum moment indicates slight enhancement.

Figure 4 shows the effect of submergence depth on reflection and transmission coefficients for a domain of $\frac{a}{h_1} = 0.2, \frac{h_1}{b} = 6, \theta = 1^0$. It is patent that increasing the submergence depth results in decreasing the reflection coefficient and increasing the transmission coefficient. It was completely predictable according to the fact that most of energy of incident waves are in the water surface. Furthermore one may conclude from Figure 4 that reflection coefficient has a linear relation with respect to the changes of non-dimensional submergence depth $\frac{s_1}{h_1}$. Also, the trend changes shows that when the submergence depth decreases, it not only results in decrement of reflection coefficient, but also leads to reduction in the range of the wave number, in which the breakwater is effective to act as a wave barrier.

It should be noted that the zero encounter angle of incident wave is singular for the present method. It is noteworthy to state that another singular condition appears where the non-dimensional submergence depth of breakwater becomes too small ($\left[\frac{s_1}{h_1}\right] < 0.01$).

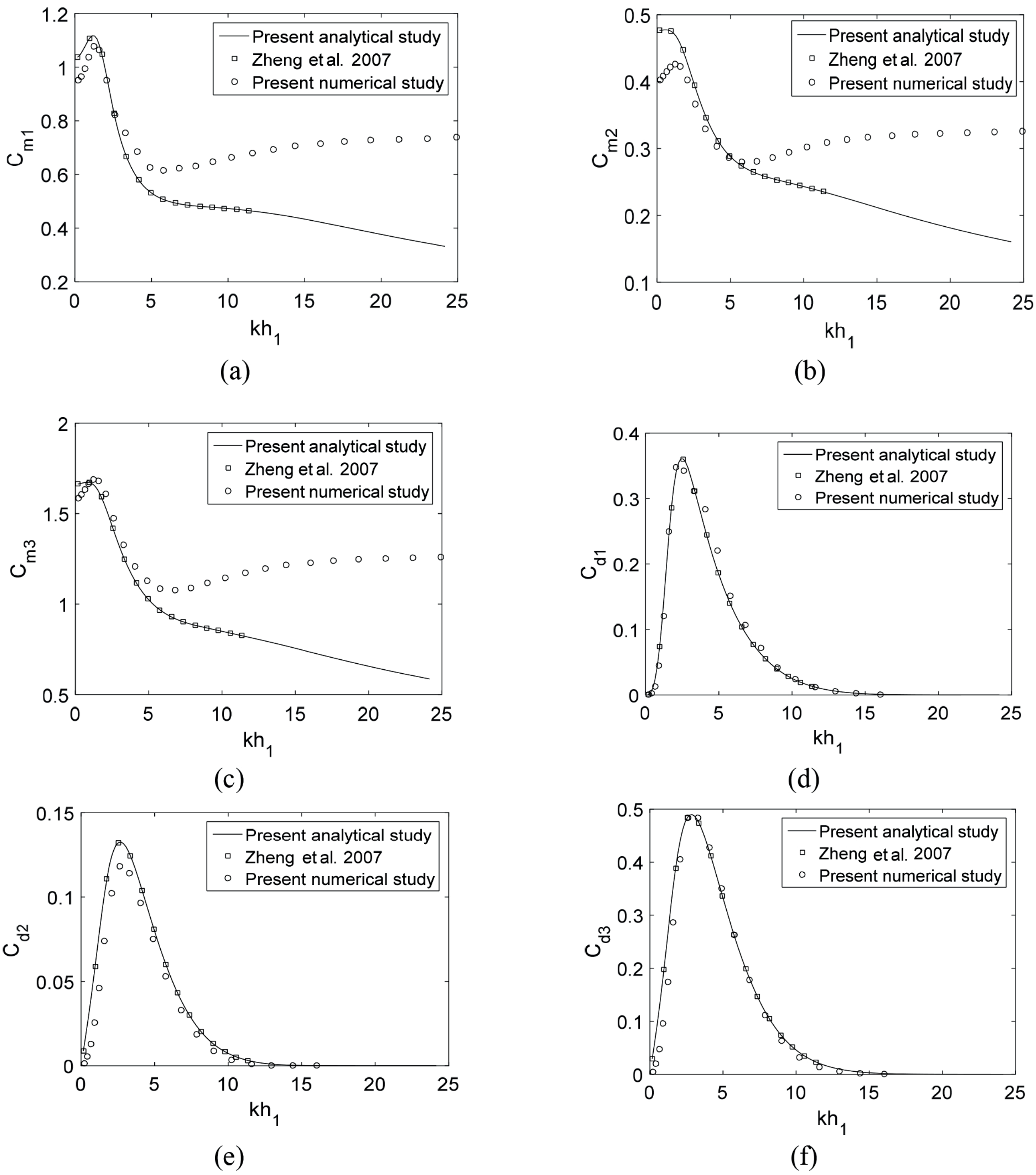


Figure 2. Added masses, damping coefficients on the submerged breakwater ($\frac{s_1}{h_1} = 0.2, \frac{a}{h_1} = 0.2, \frac{h_1}{b} = 6, \theta = 30^0$)

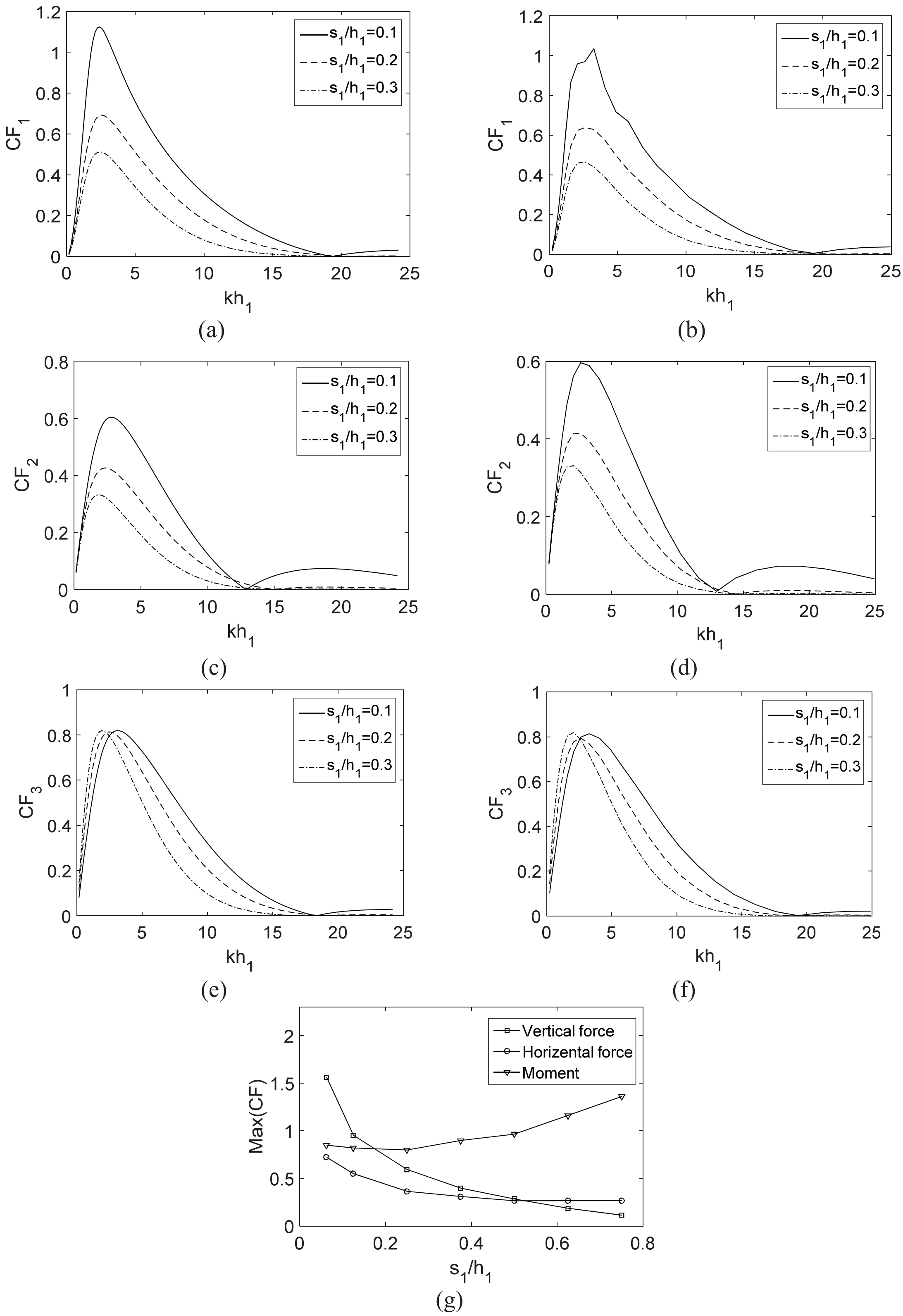
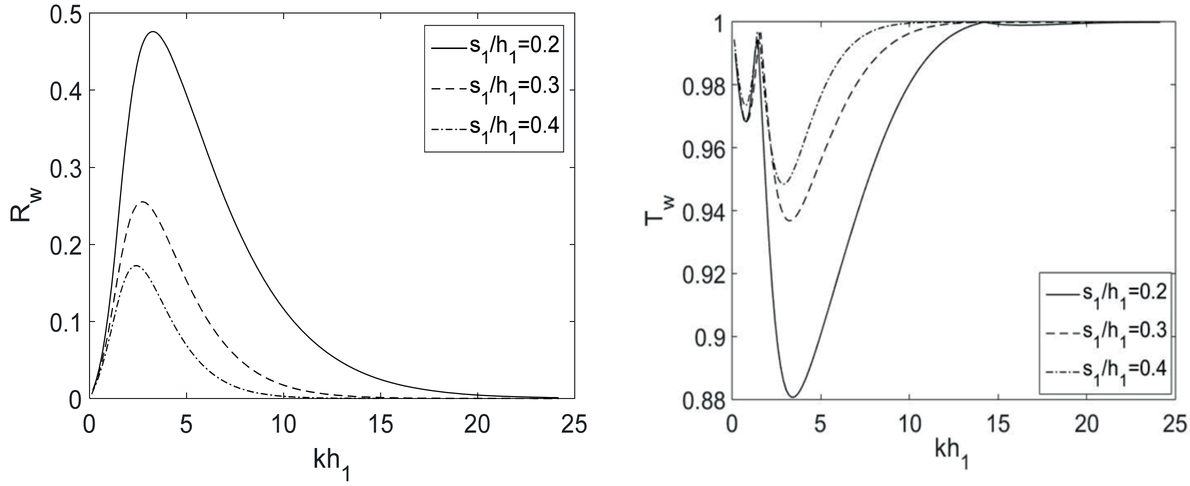


Figure 3. Effect of submergence depth on exciting forces and maximum exciting forces on submerged breakwater using analytical (a, c, e) and numerical (b, d, f) method

$(\frac{a}{h_1} = 0.2, \frac{h_1}{b} = 6, \theta = 1^0)$



(a) (b)

Figure 4. Effect of submergence depth on transmission and reflection coefficient of submerged breakwater using analytical method ($\frac{a}{h_1} = \mathbf{0.2}, \frac{h_1}{b} = \mathbf{6}, \boldsymbol{\theta} = \mathbf{1^0}$)

Conclusions

In this study, 2D submerged breakwaters with rectangular cross section were studied in infinite fluid domain of finite water depth in regular, sinusoidal waves. An analytical and numerical method in the framework of linear theory is applied to extract the hydrodynamic characteristic of the domain. Exciting forces as well as added mass and damping coefficients of the submerged breakwater were compared with previous studies for verification of the study and a good agreement was achieved. After that, a parametric study was carried out in which the effect of submergence depth on the transmission and reflection coefficients and exciting forces was studied. The results represented that when the submergence parameter increases, the reflection coefficient decreases and the transmission coefficient increases. Also, the horizontal and vertical exciting forces decrease when the submergence parameter increases. It is in compliance with the fact that the most of energy of waves are concentrated near the free surface.

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